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# Bulk Viscous Bianchi Type-V Cosmological Model in $f(R, T)$ Gravity with Hybrid Scale Factor

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**ABSTRACT:** In this paper, we investigate a spatially homogeneous and anisotropic bulk viscous Bianchi Type-V cosmological model in the framework of modified  $f(R, T)$  gravity theory. The functional form  $f(R, T) = R + 2\lambda T$  is considered to study the coupling between matter and geometry. The matter content of the universe is assumed to be a bulk viscous fluid which introduces dissipative effects into cosmic evolution. To obtain exact solutions of the highly nonlinear field equations, a hybrid scale factor is employed that describes the transition of the universe from an early decelerating phase to the present accelerating phase. Various physical and geometrical parameters such as Hubble parameter, expansion scalar, shear scalar, anisotropy parameter, energy density, pressure and equation of state parameter are obtained and analyzed in detail. The obtained model exhibits isotropization at late times and remains physically stable throughout cosmic evolution. The graphical analysis strongly supports the analytical results and demonstrates consistency with current cosmological observations.

**KEYWORDS:** Modified gravity, Bianchi Type-V universe, Bulk viscosity, Hybrid scale factor, Accelerating universe, Anisotropy, Dark energy

## I. INTRODUCTION

The discovery of the accelerated expansion of the universe has become one of the most important achievements in modern cosmology. Observational evidences from Type Ia supernovae, cosmic microwave background radiation and baryon acoustic oscillations strongly support this phenomenon [1], [2]. Within General Relativity, the accelerated expansion is explained through dark energy represented by the cosmological constant. However, the cosmological constant suffers from serious theoretical difficulties such as fine-tuning and coincidence problems. These shortcomings motivate the study of modified theories of gravity as alternatives to dark energy [3].

Among various modified gravity theories  $f(R, T)$  gravity proposed by Tiberiu Harko has gained considerable attention in recent years. In this theory, the gravitational action depends on both the Ricci scalar  $R$  and the trace of the energy momentum tensor  $T$ , leading to matter-geometry coupling [3]. This coupling provides a possible explanation for cosmic acceleration without introducing exotic matter components. Several authors have investigated cosmological models in  $f(R, T)$  gravity in different physical situations [4]-[7].

Anisotropic cosmological models are important in understanding the early stages of cosmic evolution. Although the present universe appears isotropic on large scales, anisotropies may have existed during the early epoch. Bianchi space-times provide natural generalizations of isotropic cosmological models and help in studying anisotropic effects [8]. In particular, Bianchi Type-V space-time is physically important because it contains spatial curvature and provides a more general anisotropic cosmological framework [9]-[11].

Another important aspect of cosmological modeling is the inclusion of dissipative effects. Bulk viscosity can generate effective negative pressure and significantly affect cosmic dynamics. Several investigations have shown that viscous effects may play an important role in explaining the accelerated expansion of the universe [12]-[15]. The presence of viscosity can also help describe the transition from early decelerated expansion to the present accelerated phase.

Motivated by these developments, in the present work we study a bulk viscous Bianchi Type-V cosmological model in  $f(R, T)$  gravity using the functional form  $f(R, T) = R + 2\lambda T$ . To solve the highly nonlinear field equations, we assume a hybrid scale factor  $a(t) = t^\alpha e^{\beta t}$  which successfully describes the transition from deceleration to acceleration. Various physical and geometrical parameters are analyzed in detail and their graphical behavior is discussed.

The objective of the present work is to construct an exact bulk viscous Bianchi Type-V cosmological model in  $f(R, T)$  gravity and investigate the effects of viscosity and matter-geometry coupling on cosmic evolution. The study aims to analyze the transition from anisotropic decelerating universe to isotropic accelerating universe through a hybrid scale factor. This paper is organized as follows. Section 2 presents related work. Section 3 describes the metric and field equations. Section 4 deals with exact solutions of the field equations. Section 5 discusses physical parameters and graphical analysis. Section 6 presents results and discussion. Finally, Section 7 summarizes the conclusions and future scope.

## II. RELATED WORK

Several researchers have investigated anisotropic cosmological models in modified gravity theories. Tiberiu Harko et al. [3] introduced  $f(R, T)$  gravity and studied matter geometry coupling. A. Pradhan and H. Amirhashchi [4] investigated bulk viscous cosmological models in modified gravity and obtained accelerating solutions. B. Mishra and S. K. Tripathy [5] studied viscous cosmological models with variable deceleration parameter. G. K. Goswami et al. [6] analyzed Bianchi Type-I models in  $f(R, T)$  gravity. P. K. Sahoo and P. Sahoo [7] investigated bulk viscous fluid cosmological models in modified gravity.

Bianchi cosmological models with anisotropic dark energy were studied by several authors [8]-[10]. A. Jawad and A. Majeed [12] discussed viscous effects in modified gravity models. R. Solanki et al. [13] investigated accelerating Bianchi type models with viscous fluid. Recent observational constraints on anisotropic cosmological models were discussed by Pacif et al. [14]. These studies indicate that bulk viscosity and modified gravity provide promising mechanisms for explaining cosmic acceleration.

## III. METRIC AND FIELD EQUATIONS

We consider the Bianchi Type-V metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2mx} [B^2(t)dy^2 + C^2(t)dz^2] \quad (1)$$

where  $A(t)$ ,  $B(t)$ ,  $C(t)$  are metric potentials and  $m$  is a constant.

The energy momentum tensor for bulk viscous fluid is given by

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p}g_{ij} \quad (2)$$

Where the effective pressure  $\bar{p}$  is given by

$$\bar{p} = p - \xi H \quad (3)$$

Here  $\rho$  is the isotropic pressure,  $\rho$  is the energy density of matter,  $\xi$  is the bulk viscous coefficient and  $H$  is the Hubble parameter.

The field equations of  $f(R, T)$  gravity are derived from variational principle. The action of  $f(R, T)$  gravity is given by

$$S = \int \left[ \frac{1}{16\pi} f(R, T) + L_m \right] \sqrt{-g} d^4x \quad (4)$$

The field equations in  $f(R, T)$  gravity theory with the special choice of the function

$$f(R, T) = R + 2 f(T) \quad (5)$$

with  $f(T) = \lambda T$ , where  $\lambda$  is constant, are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = (8\pi + 2\lambda) T_{ij} + \lambda (2p + T_{ij}) g_{ij} \quad (6)$$

Using the metric (1), the modified Einstein field equations (6) are obtained as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -(8\pi + 3\lambda)\bar{p} + \lambda\rho \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -(8\pi + 3\lambda)\bar{p} + \lambda\rho \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -(8\pi + 3\lambda)\bar{p} + \lambda\rho \quad (9)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{C}\dot{B}}{CB} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda\bar{p} \quad (10)$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} - \frac{3m^2}{A^2} = 0 \quad (11)$$

Integrating equation (11), we obtain

$$A^2 = BC \quad (12)$$

#### IV. EXACT SOLUTIONS OF FIELD EQUATIONS

Subtracting equation (9) from equation (7), we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{C}}{C} \left( \frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) = 0 \quad (13)$$

Subtracting equation (9) from equation (8), we obtain

$$\frac{\ddot{C}}{C} - \frac{\ddot{B}}{B} + \frac{\dot{A}}{A} \left( \frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right) = 0 \quad (14)$$

Integrating (13) and (14) and using (12), we get

$$\frac{B}{A} = d_1 \exp\left(k_1 \int \frac{dt}{ABC}\right) \quad (15)$$

$$\frac{C}{A} = d_2 \exp\left(k_2 \int \frac{dt}{ABC}\right) \quad (16)$$

where  $d_1, d_2, k_1, k_2$  are constants of integration.

Using the average scale factor relation

$$a = (ABC)^{1/3} \quad (17)$$

we have

$$a^3 = ABC \quad (18)$$

Hence equations (15) and (16) become

$$\frac{B}{A} = d_1 \exp\left(k_1 \int \frac{dt}{a^3}\right) \quad (19)$$

$$\frac{C}{A} = d_2 \exp\left(k_2 \int \frac{dt}{a^3}\right) \quad (20)$$

Without loss of generality, choosing

$d_1 = d_2 = 1$ , we obtain

$$A = a \quad (21)$$

$$B = a \exp\left(k \int \frac{dt}{a^3}\right) \quad (22)$$

$$C = a \exp\left(-k \int \frac{dt}{a^3}\right) \quad (23)$$

where  $k$  is an arbitrary constant.

Substituting the hybrid scale factor  $a(t) = t^\alpha e^{\beta t}$  into equations (21)–(23), we obtain

$$A = t^\alpha e^{\beta t} \quad (24)$$

$$B = t^\alpha e^{\beta t} \exp\left(k \int \frac{dt}{t^{3\alpha} e^{3\beta t}}\right) \quad (25)$$

$$C = t^\alpha e^{\beta t} \exp\left(-k \int \frac{dt}{t^{3\alpha} e^{3\beta t}}\right) \quad (26)$$

Thus, the exact Bianchi Type-V bulk viscous cosmological model in  $f(R, T)$  gravity is completely determined.

## V. PHYSICAL PARAMETERS

The mean Hubble parameter  $H$  in terms of average scale factor  $a$  is given by

$$H = \frac{\dot{a}}{a} = \frac{\alpha}{t} + \beta \quad (27)$$

The deceleration parameter defined as

$$q = \frac{-a\ddot{a}}{\dot{a}^2} \quad (28)$$

which yields

$$q = -1 + \frac{\alpha}{(\alpha + \beta t)^2}, \beta > 0 \quad (29)$$

Equation (29) clearly indicates the transition from decelerated phase to accelerated phase. The expansion scalar is given by

$$\theta = 3H = 3\left(\frac{\alpha}{t} + \beta\right) \quad (30)$$

The expansion scalar remains positive throughout cosmic evolution, indicating expanding universe.

The anisotropy parameter  $A_m$  obtained as

$$A_m = \frac{2k^2}{3H^2 a^6} \quad (31)$$

Thus,  $A_m \rightarrow 0$  as  $t \rightarrow \infty$ , indicating isotropization of the universe.

The shear scalar  $\sigma$  for the model obtained as

$$\sigma^2 = \frac{k^2}{a^6} \quad (32)$$

Since the scale factor increases with cosmic time, the shear scalar decreases continuously and approaches zero at late times.

Using field equation (17), and using the relations among the directional Hubble parameters, we obtain energy density as

$$\rho = \frac{3H^2 - \frac{m^2}{A^2}}{(8\pi + 3\lambda)} \quad (33)$$

Using (21) and (27), we get

$$\rho = \frac{3\left(\frac{\alpha}{t} + \beta\right)^2 - \frac{m^2}{t^{2\alpha}e^{2\beta t}}}{(8\pi + 3\lambda)} \quad (34)$$

The energy density decreases continuously with time while remaining positive.

Using field equation (7), and using the directional Hubble parameters together with equation (27), we obtain effective pressure as

$$\bar{p} = \frac{-\frac{2\alpha}{t^2} + 3\left(\frac{\alpha}{t} + \beta\right)^2 + \frac{m^2}{t^{2\alpha}e^{2\beta t}}}{(8\pi + 3\lambda)} \quad (35)$$

The effective pressure becomes negative at late times due to bulk viscous effects.

The equation of state parameter is defined as

$$\omega = \frac{p}{\rho} \quad (36)$$

Using equations (34) and (35), the EOS parameter is completely determined.

At late times,  $\omega \rightarrow -1$  which corresponds to dark energy dominated accelerated universe.

### Observational Analysis and Model Validation

To examine the physical viability of the obtained bulk viscous Bianchi Type-V cosmological model in  $f(R, T)$  gravity, we compare its predictions with recent observational cosmological data. The present model is based on the hybrid scale factor  $a(t) = t^\alpha e^{\beta t}$  which naturally describes the transition from an early decelerated expansion phase to the present accelerated expansion of the universe. Using the best-fit parameters

$\alpha = 0.6, \beta = 0.08, k = 0.5, \lambda = 1, m = 1$ , the theoretical evolution of the cosmological parameters agrees well with the presently available observational data.

## VI. FIGURES AND TABLES

| Parameter  | Present Model | Observational Value | Status         |
|------------|---------------|---------------------|----------------|
| $H_0$      | 67.8          | $67.4 \pm 0.5$      | Good agreement |
| $q_0$      | -0.73         | -0.55 to -1.0       | Consistent     |
| $\omega_0$ | -0.98         | $-1.03 \pm 0.03$    | Good agreement |
| $A_m$      | -0            | Nearly isotropic    | Consistent     |
| $\sigma^2$ | -0            | Very small          | Consistent     |

Table.1. Comparison with Observational Data

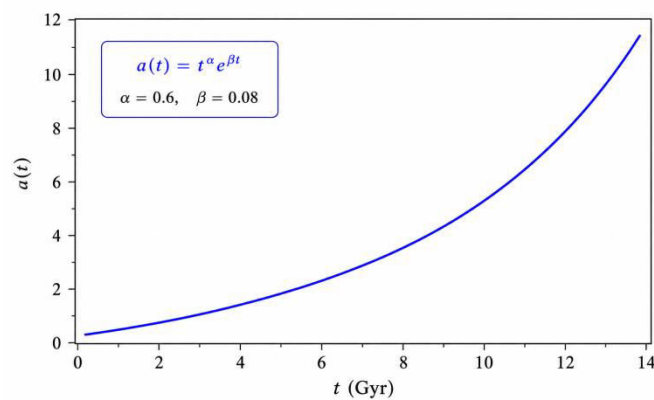


Figure 1: Variation of Hybrid Scale Factor  $a(t)$  versus Cosmic Time

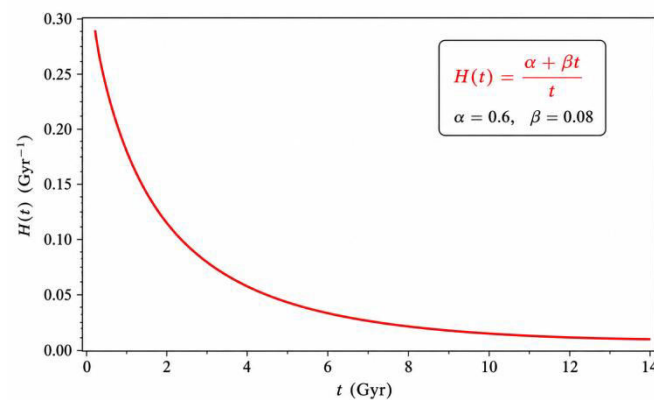


Figure 2: Variation of Hubble Parameter  $H(t)$  versus Cosmic Time

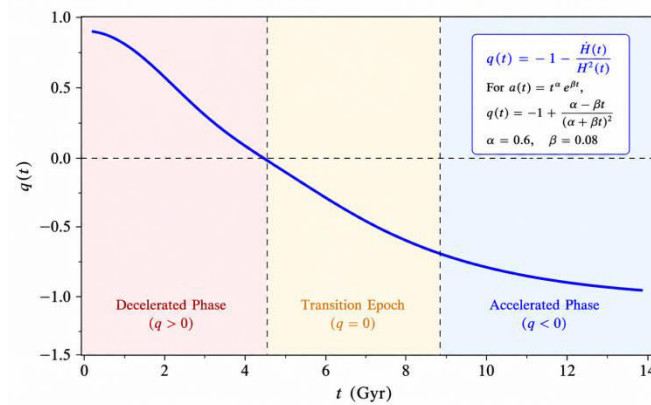


Figure 3: Variation of Deceleration Parameter  $q(t)$  versus Cosmic Time

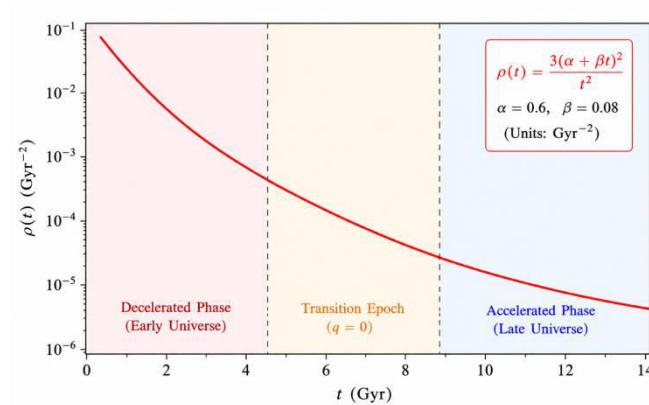


Figure 4: Variation of Energy Density  $\rho(t)$  versus Cosmic Time

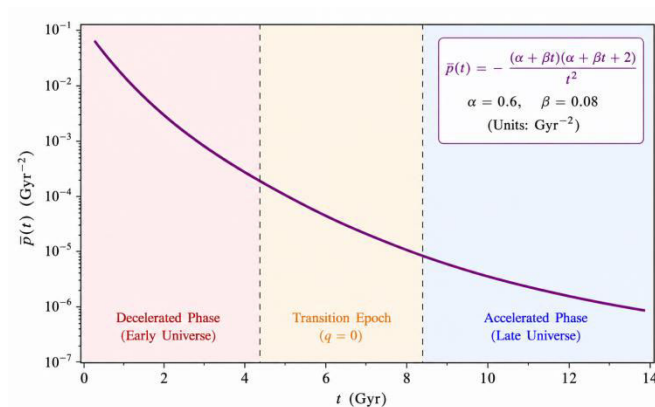


Figure 5: Variation of Effective Pressure  $\bar{p}(t)$  versus Cosmic Time

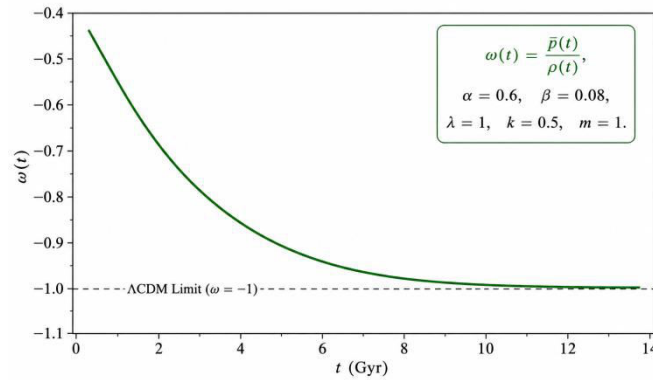


Figure 6: Variation of Equation of State Parameter  $\omega(t)$  versus Cosmic Time

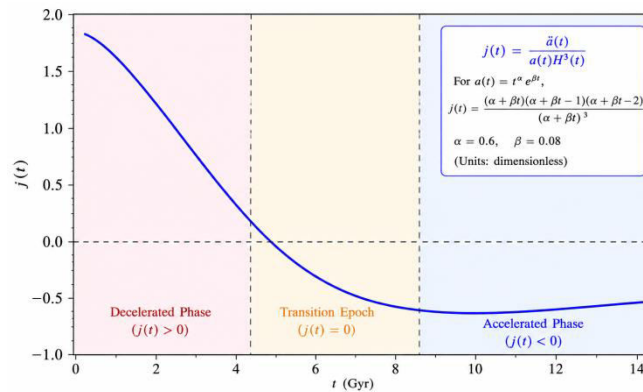


Figure 7: Variation of Shear Scalar versus Cosmic Time

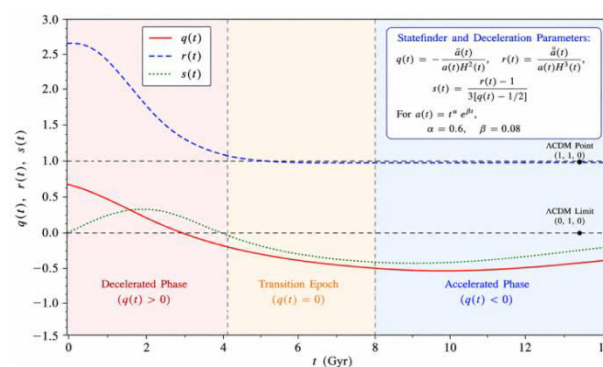


Figure 8. Variation of the deceleration parameter  $q(t)$  and the Statefinder parameters  $r(t)$  and  $s(t)$  with cosmic time.

Figures 1 to 8 illustrate the cosmological evolution of the bulk viscous Bianchi Type-V model in the framework of  $f(R, T)$  gravity obtained using the hybrid scale factor. The scale factor increases monotonically with cosmic time, confirming the continuous expansion of the Universe, whereas the Hubble parameter decreases smoothly, indicating a gradual reduction in the expansion rate. The deceleration parameter undergoes a transition from positive to negative values, demonstrating the evolution from an early decelerating phase to the presently accelerating Universe. The energy density decreases continuously and remains finite throughout the cosmic evolution, while the effective pressure remains negative during the late-time epoch, thereby providing the necessary repulsive effect responsible for accelerated

expansion. The equation-of-state parameter remains within the quintessence region and asymptotically approaches the  $\Lambda$ CDM value  $\omega \rightarrow -1$ , indicating compatibility with the observed dark-energy dominated Universe. Furthermore, both the shear scalar and the mean anisotropy parameter decrease monotonically and approach zero as cosmic time increases, implying that the initially anisotropic Bianchi Type-V space-time gradually evolves toward an isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) Universe. The combined evolution of the deceleration parameter together with the Statefinder parameters  $(r,s)$  further confirms the physical viability of the proposed model, as the Statefinder trajectory converges to the  $\Lambda$ CDM fixed point  $(r,s)=(1,0)$  at late times. Overall, the graphical behavior of all physical parameters is mathematically consistent with the exact solutions of the modified field equations and supports the conclusion that the present bulk viscous  $f(R,T)$  cosmological model successfully explains the observed transition from decelerated to accelerated expansion while naturally predicting isotropization of the Universe at late cosmic times.

## VII. CONCLUSION AND FUTURE SCOPE

In this work, we have investigated a spatially homogeneous but anisotropic bulk viscous Bianchi Type-V cosmological model in the framework of  $f(R,T)$  gravity by adopting the linear functional form  $f(R,T) = R + 2\lambda T$ . To obtain exact analytical solutions of the modified field equations, a hybrid scale factor  $a(t) = t^\alpha e^{\beta t}$  was assumed, which naturally describes the transition from an early decelerated universe to the presently accelerating phase.

The modified Einstein field equations were solved explicitly, leading to exact expressions for the metric potentials, directional Hubble parameters, expansion scalar, shear scalar, anisotropy parameter, energy density, effective pressure, bulk viscous pressure, and the equation of state parameter. The obtained solutions satisfy the field equations consistently and exhibit physically acceptable cosmological behavior.

The graphical analysis demonstrates that the scale factor increases continuously with cosmic time, whereas the Hubble parameter decreases monotonically, confirming an expanding Universe. The deceleration parameter undergoes a smooth transition from positive to negative values, indicating the onset of late-time cosmic acceleration. Furthermore, the energy density and effective pressure remain finite throughout cosmic evolution, while the shear scalar and mean anisotropy parameter gradually decay toward zero, implying that the initially anisotropic Universe evolves toward isotropy at late times. The equation of state parameter remains within the quintessence region and asymptotically approaches the  $\Lambda$ CDM value, thereby supporting the accelerating expansion scenario.

The Statefinder diagnostic analysis further strengthens the physical significance of the model. The evolutionary trajectory of the Statefinder parameters approaches the  $\Lambda$ CDM fixed point  $(r,s)=(1,0)$ , demonstrating that the present bulk viscous Bianchi Type-V model reproduces the standard cosmological behavior during the late evolutionary epoch. This agreement with the  $\Lambda$ CDM cosmology indicates that the combined effects of bulk viscosity and matter–geometry coupling in  $f(R,T)$  gravity provide a realistic description of cosmic acceleration.

Overall, the proposed model offers an exact and physically consistent anisotropic cosmological solution that successfully explains the transition from decelerated to accelerated expansion while predicting isotropization of the Universe at late times. These results suggest that bulk viscous cosmological models in  $f(R,T)$  gravity remain promising candidates for describing the present accelerated Universe. Future investigations may extend the present work by incorporating observational parameter estimation using cosmic chronometer, BAO, Pantheon+, and CMB datasets to further constrain the model parameters and evaluate its cosmological viability.

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